Exercise 7.4

Part A

1. Line 1 intersects both the x-axis and the y-axis. Line 2 intersects only the z-axis. Neither contains the origin. Must the two lines be parallel or skew, or can they intersect?

2. Find the intersection point of each of the following pairs of lines. Graph the lines and identify the intersection point.

   a. $2x + 5y + 15 = 0$
   $3x - 4y + 11 = 0$

   b. $\vec{r} = (-3, -6) + s(1, 1)$
   $\vec{r} = (4, -8) + t(1, 2)$

3. Determine whether the following pairs of lines are coincident, parallel and distinct, or neither.

   a. $\frac{x-3}{10} = \frac{y-8}{-4}$
   $\frac{x-33}{-5} = \frac{y+4}{2}$

   b. $x = 6 - 18s, y = 12 + 3s$
   $x = 8 - 6t, y = 4 + 9t$

   c. $x = 8 + 12s, y = 4 - 4s, z = 3 - 6s$
   $x = 2 - 4t, y = 2 + t, z = 6 + 2t$

   d. $\frac{x+4}{3} = \frac{y-12}{4} = \frac{z-3}{6}$
   $\frac{x}{\frac{1}{2}} = \frac{y-10}{\frac{2}{3}} = z + 5$

Part B

4. Find the intersection of each pair of lines. If they do not meet, determine whether they are parallel and distinct or skew.

   a. $\vec{r} = (-2, 0, -3) + t(5, 1, 3)$
   $\vec{r} = (5, 8, -6) + u(-1, 2, -3)$

   b. $x = 1 + t, y = 1 + 2t, z = 1 - 3t$
   $x = 3 - 2u, y = 5 - 4u, z = -5 + 6u$

   c. $\vec{r} = (2, -1, 0) + t(1, 2, -3)$
   $\vec{r} = (-1, 1, 2) + u(-2, 1, 1)$

   d. $(x, y, z) = (1 + t, 2 + t, -t)$
   $(x, y, z) = (3 - 2u, 4 - 2u, -1 + 2u)$

   e. $\frac{x-3}{4} = y - 2 = z - 2$
   $\frac{x-2}{-3} = \frac{y+1}{2} = \frac{z-2}{-1}$

5. Consider the lines $\vec{r} = (1, -1, 1) + t(3, 2, 1)$ and $\vec{r} = (-2, -3, 0) + u(1, 2, 3)$.

   a. Find their point of intersection.

   b. Find a vector equation for the line perpendicular to both of the given lines that passes through their point of intersection.

6. Show that the lines $\vec{r} = (4, 7, -1) + t(4, 8, -4)$ and $\vec{r} = (1, 5, 4) + u(-1, 2, 3)$ intersect at right angles and find the point of intersection.

7. If they exist, find the x-, y-, and z-intercepts of the line $x = 24 + 7t$, $y = 4 + t, z = -20 - 5t$.

8. Find the point at which the normal through the point $(3, -4)$ to the line $10x + 4y - 101 = 0$ intersects the line.

Part C

9. What are the possible ways that three lines in a plane can intersect? Describe them all with diagrams.

10. What are the possible ways that three lines in space can intersect? Describe them all with diagrams.

11. Find the equation of the line through the point $(-5, -4, 2)$ that intersects the line at $\vec{r} = (7, -13, 8) + t(1, 2, -2)$ at 90°. Determine the point of intersection.

12. Find the points of intersection of the line $\vec{r} = (0, 5, 3) + t(1, -3, -2)$ with the sphere $x^2 + y^2 + z^2 = 6$. Is the segment of the line between the intersection points a diameter of the sphere?

13. Find a vector equation for the line through the origin that intersects both of the lines $\vec{r} = (2, -16, 19) + t(1, 1, -4)$ and $\vec{r} = (14, 19, -2) + u(-2, 1, 2)$.

14. a. Determine the point N at which the normal through the origin intersects the line $Ax + By + C = 0$ in the xy-plane.

    b. Find the magnitude of the position vector $\overrightarrow{ON}$ of point N.

15. The common perpendicular of two skew lines with direction vectors $\overrightarrow{d_1}$ and $\overrightarrow{d_2}$ is the line that intersects both the skew lines and has direction vector $\vec{n} = \overrightarrow{d_1} \times \overrightarrow{d_2}$. Find the points of intersection of the common perpendicular with each of the lines $(x, y, z) = (0, -1, 0) + s(1, 2, 1)$ and $(x, y, z) = (-2, 2, 0) + t(2, -1, 2)$.

16. The distance between the skew lines $\vec{r} = \overrightarrow{OP} + t\overrightarrow{d_1}$ and $\vec{r} = \overrightarrow{OQ} + s\overrightarrow{d_2}$ is $|\text{Proj}(\overrightarrow{PQ} \text{ onto } \vec{n})|$ or $\frac{|\overrightarrow{PQ} \cdot \vec{n}|}{|\vec{n}|}$ where $\vec{n} = \overrightarrow{d_1} \times \overrightarrow{d_2}$.
    Find the distance between the lines

    a. $\vec{r} = (0, -2, 6) + t(2, 1, -1)$ and $\vec{r} = (0, -5, 0) + s(-1, 1, 2)$

    b. $x = 6, y = -4 - t, z = t$ and $x = -2s, y = 5, z = 3 + s$